MATHEMATICS PG
(**FINAL**)

1. The inequality $|z-4|<|z-2|$ represents the region given by

(A) $\text{Re}(z)<3$
(B) $\text{Re}(z)>3$
(C) $\text{Re}(z)>0$
(D) $\text{Re}(z)<0$

2. The set of points where the function f given by $f(x)=|2x-1|\sin x$ is differentiable is

(A) R
(B) $R\setminus\left\{\frac{1}{2}\right\}$
(C) $(0,\infty)$
(D) $(-\infty,\infty)$

3. The radius of convergence R of the series $\sum\frac{\log n}{n}x^n$ is equal to

(A) 1
(B) ∞
(C) 2
(D) 3

4. If $z=x+iy$ is a complex number, then $\left|e^{z^2}\right|=e^{|z|^2}$ is true

(A) for all $z\in\mathbb{C}$
(B) if and only if $y=0$
(C) if and only if $x=0$
(D) only when $z=0$

5. The number of group homomorphisms $\pi:\mathbb{Z}\to\mathbb{Z}$ is

(A) one
(B) two
(C) even
(D) ∞

6. Which of the following function is not uniformly continuous on $(0,1)$?

(A) $\frac{1}{x^2}$

(B) x^2

(C) $\frac{\sin x}{x}$

(D) $\sin x$

7. If $f(z) = \cos x\left(\cosh y + a \sinh y\right) + i \sin x\left(\cosh y + b \sinh y\right)$ satisfies the C-R equation, then

(A) $a = 1,\ b = -1$

(B) $a = i,\ b = -1$

(C) $a = i,\ b = -i$

(D) $a = -1,\ b = -1$

8. If the eigen value of $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ are $-2, 3, 6,$ then the eigen values of A^T are

(A) $\frac{-1}{2}, \frac{1}{3}, \frac{1}{6}$

(B) $-2, 3, 6$

(C) $2, -3, -6$

(D) $\frac{1}{2}, \frac{-1}{3}, \frac{-1}{6}$

9. $\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \ldots + \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}}\right) =$

(A) $\frac{1}{\sqrt{2}}$

(B) $\sqrt{2}$

(C) $\sqrt{2}+1$

(D) $\frac{1}{\sqrt{2}+1}$

10. Which one of the following is **WRONG**?

(A) every Cauchy sequence is convergent
(B) every Cauchy sequence in R is convergent
(C) every Cauchy sequence in R is bounded
(D) a sequence of real numbers is unbounded

11. Given $\overline{x} = 25,\ \overline{y} = 22,\ \sigma_x = 4,\ \sigma_y = 5,\ \gamma = 0.8.$ Then the regression line of y on x is

(A) $x + y + 3 = 0$
(B) $x - y + 3 = 0$
(C) $x - y - 3 = 0$
(D) $-x + y + 3 = 0$

12. The derivative of $f(\tan x)$ with respect to $g(\sec x)$ at $x = \frac{\pi}{4}$ where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$ is

(A) $\frac{1}{\sqrt{3}}$
(B) $\sqrt{3}$
(C) $\sqrt{2} + 1$
(D) $\frac{1}{\sqrt{2}}$

13. A metric space is totally bounded if and only if every sequence has a

(A) subsequence
(B) bounded subsequence
(C) Cauchy subsequence
(D) infinite subsequence

14. The order of the coset $\overline{4}$ in the quotient ring $\frac{\mathbb{Z}}{6\mathbb{Z}}$ is

(A) 6
(B) 5
(C) 3
(D) 1

15. The equation $z\,\overline{z}+i\,\overline{z}-i\,z-3=0$ is

(A) a straight line
(B) a circle
(C) an ellipse
(D) a pair of straight lines

16. Let G be a group and $o(G) < 400$. If G has subgroups of order 45 and 75, then $O(G)$ is equal to

(A) 135
(B) 150
(C) 225
(D) 300

17. Let the matrices $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} i & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} -i & 0 \\ 0 & 0 \end{pmatrix}$ form a group with respect to matrix multiplication. Then which of the statement about the group is **TRUE**?

(A) The group has no element of order 4
(B) The group has an element of order 3
(C) The group is non-commutative
(D) There exists a non-identity element which is its own inverse

18. The variance of the first n natural numbers is

(A) n^2-1

(B) $\dfrac{n(n+1)}{12}$

(C) $\dfrac{n^2-1}{12}$

(D) $\dfrac{n(n-1)}{2}$

19. The function $f(z)=xy+iy$ is

(A) nowhere analytic
(B) analytic every where
(C) analytic only at origin
(D) analytic except at the origin

20. With usual notations, for any graph G

(A) $\kappa(G) \le \lambda(G) \le \delta(G)$

(B) $\kappa(G) \le \delta(G) \le \lambda(G)$

(C) $\lambda(G) \le \delta(G) \le \kappa(G)$

(D) $\lambda(G) \le \kappa(G) \le \delta(G)$

21. The order of an element 'a' in a group G is 30. Then the order of a^{18} is equal to

(A) 3
(B) 5
(C) 6
(D) 8

22. Angle of intersection between two polar curves given by $r = a(1+\sin\theta)$ and $r = a(1-\sin\theta)$ is

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) 0

23. The value of $\int_C \frac{z+2}{z} dz$ where C is the semicircle $z = 2e^{i\theta}$, $0 \le \theta \le \pi$ is

(A) $-4+4\pi i$
(B) $-4+2\pi i$
(C) $-4-2\pi i$
(D) $-4-4\pi i$

24. Let $f: G \to H$ be a group homomorphism with kernel K. If the orders of G, H and K are 75, 45 and 15 respectively, then the order of the image $f(G)$ is

(A) 3
(B) 5
(C) 15
(D) 45

25. A noncyclic abelian group all of whose proper subgroups are cyclic is

(A) $\mathbb{Z}_2$

(B) S_3

(C) $\mathbb{Z}_2 \times \mathbb{Z}_2$

(D) $\mathbb{Z}_2 \times S_4$

26. What is the envelope of straight lines given by $x\cos b + y\sin b = a\sec b,$ where b is the parameter?

(A) $y^2 + 4a(a-x) = 0$

(B) $y + 4ax = 0$

(C) $y + 4a(a-x) = 0$

(D) $y^2 = 4a(a-x)$

27. $\int_{-\infty}^{0} x^5 . e^x dx =$

(A) 1
(B) 199
(C) –5!
(D) 5!

28. The set $A = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in \mathbb{R} \right\}$ with usual addition and multiplication of matrices is

(A) a commutative ring but not an integral domain
(B) an integral domain
(C) a field
(D) a non-commutative ring

29. Let $a_n = \frac{n!}{n^n}$. Then (a_n) converges to

(A) 1

(B) $\frac{1}{e}$

(C) 0

(D) e

30. The value of $\Delta^{10}\left[(1-x)\left(1-1x^2\right)\left(1-3x^3\right)\left(1-4x^4\right)\right]$ is equal to

(A) 24
(B) 10!
(C) $24! \times 10!$
(D) $24 \times 10!$

31. The length of one arc of the cycloid $x=(\theta-\sin\theta),\ \ y=a(1+\cos\theta)$ is

(A) a
(B) $4a$
(C) $8a$
(D) $2a$

32. Let x, y, z be three vectors in a vector space V such that $x+y+z=0$ and $U=\langle x, y\rangle$ and $W=\langle y, z\rangle$. Then

(A) $U \subset W$
(B) $W \subset U$
(C) $U = W$
(D) $U \cap W = \{0\}$

33. Let R be the region bounded by x axis, ordinate $x=2a$ and the curve $x^2=4ay$. Then $\iint_R xy\,dy\,dx$

(A) $\frac{a^4}{3}$
(B) $\frac{a^4}{6}$
(C) $\frac{a^3}{3}$
(D) $\frac{a^2}{6}$

34. The value of $\frac{2}{5}+\frac{4}{5^2}+\frac{2}{5^3}+\frac{4}{5^4}+...$ is equal to

(A) 1

(B) $\frac{12}{7}$

(C) 0

(D) $\frac{7}{12}$

35. Let T be a linear transformation on a finite dimensional vector space V. Then

(A) Rank (T) = Nullity (T)
(B) Rank (T) = dim (V)
(C) Rank (T) > dim (V)
(D) Rank (T) $\leq$ dim (V)

36. The value of $\int_C \tan z \, dz$, where C is $|z| = 2,$ is

(A) $-2\pi i$

(B) $4\pi i$

(C) $2\pi i$

(D) $-4\pi i$

37. For any group G, let Aut(G) denote the group of automorphisms of G. Which of the following is **TRUE**?

(A) If G is finite, then Aut(G) is finite
(B) If G is cyclic, then Aut(G) is cyclic
(C) If G is infinite, then Aut(G) is infinite
(D) If Aut(G) is isomorphic to Aut(H), where G and H are two groups, then G is isomorphic to H

38. Let M and N be matrices such that $MN = M$ and $NM = N$. Then $M^2N^2 =$

(A) MN
(B) M
(C) N
(D) $-MN$

39. Consider the metric subspace $\mathbb{N}$ of the real line $\mathbb{R}$ with usual metric. Then every

(A) subset of $\mathbb{N}$ is open in $\mathbb{N}$
(B) singleton set is closed in $\mathbb{N}$
(C) open set in $\mathbb{N}$ is an open interval
(D) closed set in $\mathbb{N}$ is a closed interval

40. The $p-r$ equation of $r = a\sin\theta$ is

(A) $ap = \sqrt{r}$

(B) $ap = r^2$

(C) $ap = -r^2$

(D) $ap = \dfrac{1}{r^2}$

41. The series $\sum \dfrac{(-1)^n x^n}{n}$ converges if x belongs to

(A) $(-1,1]$
(B) $(1,1]$
(C) $(-1,0]$
(D) $(0,1)$

42. Let A be a 4×4 invertible real matrix. Which of the following is **NOT** necessarily **TRUE**?

(A) The rows of A form a basis of $\mathbb{R}^4$
(B) Null space of A contains only the 0 vector
(C) A has 4 distinct eigen values
(D) Image of the linear transformations $x \to Ax$ on $\mathbb{R}^4$ is $\mathbb{R}^4$

43. $\lim_{n\to\infty}\left(\frac{1}{\sqrt{2n^2+1}}+\frac{1}{\sqrt{2n^2+2}}+\ldots+\frac{1}{\sqrt{2n^2+n}}\right)=$

(A) $\frac{1}{2}$

(B) $\frac{1}{\sqrt{2}}$

(C) 1

(D) ∞

44. The order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 5 & 4 & 1 & 3 & 2 \end{pmatrix}$ in the symmetric group S_{12} is

(A) 2
(B) 3
(C) 4
(D) 12

45. The value of $\frac{1}{2\pi i}\int_C \frac{f'(z)}{f(z)}dz$, where C is $|z|=4$ and $f(z)=\frac{(z^2+1)}{(z^2+2z+a)^2}$ is

(A) -2
(B) 0
(C) 2
(D) 1

46. A solution of the differential equation $\frac{dy}{dx}=e^{x+y}+x^2e^y$ is

(A) $e^x-e^y+\frac{y^3}{3}=C$

(B) $e^x+e^y+\frac{x^3}{3}=C$

(C) $e^x+e^{-y}+\frac{x^3}{3}=C$

(D) $e^x+e^{-y}+\frac{y^3}{3}=C$

47. If $2y+3x-31=0$ represents the regression line of y on x, the standard deviation of x is 5 and the correlation coefficient is -0.5. Then the standard deviation of y is

(A) $\sqrt{15}$

(B) 15

(C) $\frac{15}{2}$

(D) $\frac{2}{15}$

48. The order of the group $U(20)$ where $U(n)$ is the group of units of $\mathbb{Z}_n$, is

(A) 6
(B) 8
(C) 10
(D) 16

49. Which of the following matrix has the same row space as the matrix $\begin{bmatrix} 4 & 8 & 4 \\ 3 & 6 & 1 \\ 2 & 4 & 0 \end{bmatrix}$?

(A) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

50. If G is a (p,q)-planar graph in which every face is an n-cycle, then q is equal to

(A) $n(p-2)$

(B) $\frac{n}{n-2}$

(C) $\frac{n(p-2)}{n-2}$

(D) $\frac{np-2}{n-2}$

51. Let $M(\mathbb{R})$ be the ring of $n \times n$ matrices over $\mathbb{R}$. Which of the following is **TRUE** for every $n \geq 2$?

(A) There exist matrices $A, B \in M_n(\mathbb{R})$ such that $AB - BA = I_n$, where I_n denotes the identity $n \times n$ matrix

(B) If $A, B \in M_n(\mathbb{R})$ and $AB = BA$, then A is diagonalizable over $\mathbb{R}$ and if and only if B is diagonalizable over $\mathbb{R}$

(C) AB and BA have same minimal polynomial

(D) If $A, B \in M_n(\mathbb{R})$ and AB and BA have same eigen values in $\mathbb{R}$

52. The general solution of the given differential equation $(8x+7)\frac{dy}{dx} + 2y = x$ is

(A) $c_1\left(\frac{1}{8x+7}\right)^{\frac{1}{4}} + \frac{8x+7}{40} - \frac{5}{16}$

(B) $c_1\left(\frac{1}{8x+7}\right)^{\frac{1}{4}} + \frac{8x+7}{80} - \frac{7}{16}$

(C) $c_1\left(\frac{1}{8x+7}\right)^{\frac{1}{4}} + \frac{8x+7}{40} - \frac{7}{8}$

(D) $c_1\left(\frac{1}{8x+7}\right)^{\frac{1}{4}} + \frac{8x+7}{40} - \frac{7}{16}$

53. The image of the circle $|z-3i|=3$ under the map $w=\dfrac{1}{z}$ is

(A) the real axis in the w-plane
(B) the circle passing through origin
(C) the straight line in the w-plane
(D) a straight line in the w-plane passing through origin

54. The series $\sum_{n=1}^{\infty}\dfrac{(n+1)x^n}{n^3}$; $x>0$, is

(A) convergent if $x>1$
(B) divergent if $x<1$
(C) convergent if $x\le 1$ and divergent if $x>1$
(D) convergent if $x>1$ and divergent if $x<1$

55. The automorphism group of $\mathbb{Z}$ is isomorphic to

(A) S_3
(B) $\mathbb{Z}_3$
(C) $\mathbb{Z}_2$
(D) $\mathbb{Z}$

56. The sequence $\{s_n\}$ defined by $s_{n+1}=\sqrt{7+s_n}$, $s_1=\sqrt{7}$ converges to a

(A) positive root of $x^2-x-7=0$
(B) negative root of $x^2-x-7=0$
(C) positive root of $x^2+x-7=0$
(D) negative root of $x^2+x-7=0$

57. The radius of the curvature of $y=\sin x$ at $x=\dfrac{\pi}{2}$ is

(A) -1
(B) 1
(C) 0
(D) $\dfrac{\pi}{2}$

58. Suppose φ is a non-constant and a non-identity group homomorphism from the group of S_4 to $\mathbb{Z}_2$. Then $\ker\varphi$ is

(A) S_3

(B) $\mathbb{Z}_3$

(C) $\mathbb{Z}_4$

(D) A_4

59. If a, b be two real numbers with $a > 0$ and $b > 0$, then there exists a positive integer n such that

(A) $na > b$
(B) $na < b$
(C) $na = b$
(D) $na \neq b$

60. If $0 < a < b$, then $\lim_{n\to\infty}\left(\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}\right)$ is equal to

(A) 0
(B) a
(C) b
(D) a/b

61. The vector $\dfrac{\vec{r}}{|\vec{r}|^3}$ is

(A) only solenoidal
(B) only irrotational
(C) both solenoidal and irrotational
(D) neither solenoidal nor irrotational

62. The bilinear transformation which takes the points $z = -1, 0, 2$ into the points $w = 0, i, -i$ respectively is

(A) $w = i\dfrac{z-2}{2-3i}$

(B) $w = i\dfrac{z+2}{2-3i}$

(C) $w = i\dfrac{z+2}{2+3i}$

(D) $w = i\dfrac{z-2}{2+3i}$

63. Let V be the set of all polynomials of degree $\leq n$ in $\mathbb{R}[x]$. Then the dimension of V is

(A) 1
(B) n
(C) $n-1$
(D) $n+1$

64. If $f(x) = ab^{cx}$, then $\Delta f(x)$ is equal to

(A) $ab^{cx}\left(b^{ch}-1\right)$

(B) $ab^{cx}\left(b^{h}-1\right)$

(C) $ab^{cx}\left(a^{ch}-1\right)$

(D) $ab^{cx}\left(a^{h}-1\right)$

65. Area enclosed by the curve $\pi\left[4\left(x-\sqrt{2}\right)^2+y^2\right]=8$ is

(A) 16
(B) 8
(C) 4
(D) 2

66. If $\vec{p} = i - 2j + 3k$ and $\vec{q} = 3i + 3j + k$, then $(\vec{p} - \vec{q})^2$ is equal to

(A) $\vec{p} - \vec{q}$
(B) $\vec{p} + \vec{q}$
(C) $|\vec{p}|^2 - |\vec{q}|^2$
(D) $(\vec{p} + \vec{q})^2$

67. The value of the line integral $\int_C \left(2xy^2 dx + 2x^2 y dy + dz\right)$ along a path joining the origin and the point $(1,1,1)$ is

(A) 0
(B) 2
(C) 4
(D) 6

68. Automorphism group of the group $\left(\mathbb{Z}_8, +_8\right)$ is isomorphic to

(A) Klein's 4-group
(B) $\mathbb{Z}_3$
(C) $\mathbb{Z}_4$
(D) $\mathbb{Z}_2$

69. The residue of $f(z) = \dfrac{ze^z}{(z-1)^3}$ at $z = 1$ is

(A) $\dfrac{e}{2}$
(B) $\dfrac{3e}{2}$
(C) $\dfrac{3}{2}$
(D) e

70. The largest negative integer which satisfies the inequality $\frac{x^2-1}{(x-2)(x-3)} > 0$ is

(A) -4
(B) -3
(C) -1
(D) -2

71. If the roots of the equation $9x^2 + 4ax + 4 = 0$ are imaginary, then

(A) $a \in (-3,3)$
(B) $a \in (-\infty,-3) \cup (3,\infty)$
(C) $a \in (2,3)$
(D) $a \in (3,\infty)$

72. The number of maximal ideals in $\left(\mathbb{Z}_8, +_8, \cdot_8\right)$ is

(A) 1
(B) 6
(C) 2
(D) 4

73. The characteristic roots of $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ are

(A) $-1, -1$
(B) 1, 1
(C) $\cos\theta \pm \sin\theta$
(D) $\cos\theta \pm i\sin\theta$

74. Every group of order 33 is

(A) abelian but not cyclic
(B) cyclic but not abelian
(C) non-abelian
(D) cyclic

75. $\lim_{x \to 0}\left[\left(3x+\frac{1}{x}\right)^2-\left(2x-\frac{1}{x}\right)^2\right]=$

(A) 36
(B) 10
(C) 3
(D) 2

76. The number of points of intersection of $y = x$ and $y = ke^x$, where $k \leq 0$, is

(A) ∞
(B) k
(C) 2
(D) 1

77. The value of $\sum_{n=1}^{\infty}\frac{1}{4n^2-1}$ is

(A) $\frac{1}{2}$
(B) 1
(C) 0
(D) -1

78. The mean of four numbers is 37. The mean of the smallest three of them is 34. If the range of the data is 15, then the mean of the largest three is

(A) 41
(B) 38
(C) 40
(D) 39

79. If $u(x, y) = ax^2 - y^2 + xy$ is harmonic, then

(A) $a = -1$
(B) $a = 1$
(C) $a = 0$
(D) $a = 2$

80. $\lim_{n\to\infty} \frac{1}{n}\left(1+2^{\frac{1}{2}}+3^{\frac{1}{3}}+...+n^{\frac{1}{n}}\right)=$

(A) 0
(B) 2
(C) 4
(D) 1

81. Let $f:(-1,1)\to\mathbb{R}$ be a differentiable function with $f(0)=-1$ and $f'(0)=1$. Let $g(x)=\left[f\left(2f(x)+2\right)\right]^2$. Then $g'(0)=$

(A) -4
(B) 0
(C) 2
(D) 4

82. The number of vertices in a polyhedron which has 30 edges on 12 faces, is

(A) 12
(B) 15
(C) 20
(D) 24

83. The function $f(x)=\tan x - x$

(A) increases in $(-\infty,\infty)$
(B) decreases in $(-\infty,\infty)$
(C) never decreases
(D) never increases

84. The function $f(z)=|z|^2$ is differentiable

(A) in the unit disc $|z|=1$
(B) at $z=0$
(C) whole complex plane
(D) in the straight line $z+\overline{z}=4$

85. The greatest value of the function $f(x) = xe^{-x}$ in the interval $[0, \infty)$ is

(A) e

(B) $\frac{1}{e}$

(C) 0

(D) ∞

86. The moment generating function of a standard normal variate is

(A) $e^{\mu t + \frac{t^2\sigma^2}{2}}$

(B) $e^{\frac{\mu t}{2}}$

(C) $e^{\frac{t}{2}}$

(D) $e^{\frac{t^2}{2}}$

87. $\int \tan\left(\sin^{-1} x\right) dx =$

(A) $-\sqrt{1-x^2} + c$

(B) $-\sqrt{1+x^2} + c$

(C) $\sqrt{x^2} + c$

(D) $-\sqrt{x^2} + c$

88. In $V_3(R)$, let $S = L\{(1,1,1)\}$ and $T = L\{(-1,-1,-1)\}$. Then $\dim(S \cap T)$ is

(A) 2
(B) 1
(C) 0
(D) 3

89. The zero of $f(z) = z^2 \sin z$ at $z = 0$ is of order

(A) > 3
(B) 2
(C) 3
(D) 1

90. Area bounded by the curve $y = \log_e x,\ x = 0,\ y \le 0$ and x-axis is

(A) 0
(B) 1
(C) 2
(D) ∞

91. The number of roots of the equation $|x| = x^2 + x - 4$ is

(A) 0
(B) 1
(C) 2
(D) 4

92. Particular integral of the differential equation $x^2 \dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} + 2y = e^x$ is

(A) $x^{-2}e^x$
(B) x^2e^x
(C) $x^{-2}e^{-x}$
(D) x^2e^{-x}

93. Let $f(x) = \left(\dfrac{x^a}{x^b}\right)^{a+b} \cdot \left(\dfrac{x^b}{x^c}\right)^{b+c} \cdot \left(\dfrac{x^c}{x^a}\right)^{c+a}$. Then $\dfrac{dy}{dx} =$

(A) x^{abc}
(B) $x^{a=b+c}$
(C) 1
(D) 0

94. The number of points of discontinuity of the function $f(x) = \dfrac{1}{\log|x|}$ is

(A) 3
(B) 2
(C) 1
(D) 0

95. The equations of common tangents to $y^2 = 4ax$ and $(x+a)^2 + y^2 = a^2$ are

(A) $y = \left(\dfrac{x}{\sqrt{3}} + a\right)$

(B) $y = \pm\left(\sqrt{3} + \dfrac{a}{\sqrt{3}}\right)$

(C) $y = \pm\left(\dfrac{x}{\sqrt{3}} + \sqrt{3}a\right)$

(D) $y = \left(\sqrt{3} - \dfrac{a}{\sqrt{3}}\right)$

96. If M is a discrete metric space, then the open ball $B(a,2)$ is

(A) $\{a\}$
(B) M
(C) ϕ
(D) $\{2\}$

97. The point of intersection of the line $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z+3}{4}$ with plane $2x+4y-z+1=0$ is

(A) $\left(\dfrac{10}{3}, \dfrac{3}{2}, \dfrac{5}{3}\right)$

(B) $\left(\dfrac{10}{3}, \dfrac{-3}{2}, \dfrac{5}{3}\right)$

(C) $\left(\dfrac{10}{3}, \dfrac{5}{3}, \dfrac{3}{2}\right)$

(D) $\left(\dfrac{10}{3}, \dfrac{-3}{2}, \dfrac{-5}{3}\right)$

98. The value of $\Delta \log f(x)$ is

(A) $\log\left[1-\Delta f(x)\right]$

(B) $\log\left[1+\Delta f(x)\right]$

(C) $\log\left[1+\frac{\Delta f(x)}{f(x)}\right]$

(D) $\log\left[1-\frac{\Delta f(x)}{f(x)}\right]$

99. The values of k for which the number of distinct common normals of $(x-2)^2 = 4(y-3)$ and $x^2+y^2-2x-ku-c=0$, $(c>0)$ is 3, lie in

(A) $(2,\infty)$

(B) $(4,\infty)$

(C) $(2,4)$

(D) $(10,\infty)$

100. The value of $\int\limits_{|z|=2} \frac{z}{(9-z^2)(z+i)}dz$ is

(A) π

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{5}$

(D) $\frac{\pi}{3}$

101. A point through which three normals of parabola $y^2 = 4ax$ are passing, two of which are making angles α and β with x-axis, where $\tan\alpha \tan\beta = 2$, lies on the curve

(A) $y\left(y^2 - 2ax\right) = 0$

(B) $y\left(y^2 + 2ax\right) = 0$

(C) $y\left(y^2 - 4ax\right) = 0$

(D) $y\left(y^2 - ax\right) = 0$

102. The eccentricity of an ellipse whose pair of conjugate diameters are $y = x$ and $3y = -2x$, is

(A) $\frac{2}{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{3}$

(D) $\frac{2}{\sqrt{3}}$

103. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then

(A) $a > 0, b > 0$
(B) $a > 0, b < 0$
(C) $a = 0, b \neq 0$
(D) $a \neq 0, b = 0$

104. If g is the inverse of f and $f'(x) = \frac{1}{1+x^3}$, then $g'(x)$ is equal to

(A) $1 + [g(x)]^3$

(B) $\frac{-1}{2x^2}$

(C) $\frac{1}{2\left(1+x^2\right)}$

(D) $2\left(1+x^2\right)$

105. The series $\frac{1}{3}x+\frac{1\cdot 2}{3\cdot 5}x^2+\frac{1\cdot 2\cdot 3}{3\cdot 5\cdot 7}x^3+\ldots$ converges, if

(A) $x<2$
(B) $x=2$
(C) $x=0$
(D) $x>0$

106. If $f(x)=x^n$, then the value of $f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}+\ldots+(-1)^n\frac{f^n(1)}{n!}$ is

(A) 2^n
(B) 2^{n-1}
(C) 0
(D) 1

107. If the function y defined by the equation $xy - \log y = 1$ satisfies $x\left(yy''+y'^2\right)-y''+kyy'=0,$ then the value of k is

(A) -3
(B) 3
(C) 1
(D) -1

108. Let f be a differentiable function satisfying $f(x)+f(y)+f(z)+f(x)f(y)f(z)=14$ for all $x, y, z \in R.$ Then

(A) $f'(x)<0$ for all $x\in R$
(B) $f'(x)>0$ for all $x\in R$
(C) $f'(x)\neq 0$ for all $x\in R$
(D) $f'(x)=0$ for all $x\in R$

109. $\lim_{x\to 0}\left(\frac{1}{x}-\cot x\right)$ is equal to

(A) -1
(B) 0
(C) 1
(D) ∞

110. The angle between the plane $2x - y + z = 6$ and $x + y + 2z = 7$ is

(A) 45°
(B) 90°
(C) 30°
(D) 60°

111. If $f(x) = |x|^{|\tan x|}$, then $f'\left(\frac{-\pi}{6}\right)$ is equal to

(A) $\left(\frac{\pi}{6}\right)^{1/\sqrt{3}} \left\{\frac{2\sqrt{3}}{\pi} - \frac{4}{3}\log\frac{6}{\pi}\right\}$

(B) $\left(\frac{\pi}{6}\right)^{1/\sqrt{3}} \left\{\frac{-2\sqrt{3}}{\pi} + \frac{4}{3}\log\frac{6}{\pi}\right\}$

(C) $\left(\frac{\pi}{6}\right)^{1/\sqrt{3}} \left\{\frac{2\sqrt{3}}{\pi} + \frac{4}{3}\log\frac{6}{\pi}\right\}$

(D) $\left(\frac{\pi}{6}\right)^{1/\sqrt{3}} \left\{\frac{-2\sqrt{3}}{\pi} - \frac{4}{3}\log\frac{6}{\pi}\right\}$

112. $\lim_{n\to\infty} \prod_{r=3}^{n} \frac{r^3 - 8}{r^3 + 8} =$

(A) $\frac{7}{2}$

(B) $\frac{2}{7}$

(C) 1

(D) −1

113. For $\mathbb{R}$ with usual metric, interior of $\mathbb{Q}$ is

(A) $\mathbb{R} \setminus \mathbb{Q}$
(B) $\mathbb{R}$
(C) set of irrationals
(D) the empty set

114. In a Poisson distribution, if $P\left(x=0\right)=k,$ then the variance is

(A) e^{k}

(B) e^{-k}

(C) $\log_{e} k$

(D) $\log_{e}\left(\frac{1}{k}\right)$

115. If $f(x)=\begin{cases} ax^2+1, & x\leq 1 \\ x^2+ax+b, & x>1 \end{cases}$ is differentiable at $x=1,$ then

(A) $a=1, b=1$

(B) $a=1, b=0$

(C) $a=2, b=0$

(D) $a=2, b=1$

116. $\lim\limits_{x\to\infty}\left(\frac{2x^2+3}{2x^2+5}\right)^{8x^2+3} =$

(A) e^{8}

(B) e^{-8}

(C) e^{4}

(D) e^{-4}

117. Let $f\,'(\sin x)<0$ and $f\,''(\sin x)>0$ for all $x\in\left(0,\frac{\pi}{2}\right)$ and $g(x)=f(\sin x)+f(\cos x),$ then $g(x)$ is decreasing in

(A) $\left(0,\frac{\pi}{4}\right)$

(B) $\left(0,\frac{\pi}{2}\right)$

(C) $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

(D) $\left(\frac{\pi}{6},\frac{\pi}{2}\right)$

118. $\left| \int_{10}^{19} \frac{\sin x dx}{1+x^8} \right|$ is less than

(A) 10^{-7}
(B) 10^{-11}
(C) 10^{-10}
(D) 10^{-9}

119. In a discrete metric space, the only connected subsets are

(A) finite sets
(B) whole sets
(C) singleton sets
(D) all subsets

120. If the ordinate $x = a$ divides the area bounded by x-axis, part of the curve $y = 1 + \frac{8}{x^2}$ and the ordinate $x = 2$, $x = 4$ into two equal parts, then 'a' is equal to

(A) $\sqrt{2}$
(B) $2\sqrt{2}$
(C) $3\sqrt{2}$
(D) 2

121. The centre of curvature of $y = x^2$ at origin is

(A) $(0,0)$
(B) $\left(0, \frac{1}{2}\right)$
(C) $(-1,0)$
(D) $\left(0, \frac{1}{4}\right)$

122. A curve pass through the point (0, 1) and the gradient at (x, y) on it is $y(xy - 1)$. The equation of the curve is

(A) $y(x - 1) = 1$
(B) $y(x + 1) = 1$
(C) $x(y - 1) = 1$
(D) $x(y + 1) = 1$

123. Let $S = \{(x, y, 0) / x, y \in \mathbb{R}\} \subseteq V_3(\mathbb{R})$ with standard inner product. Then $S^{\perp}$ is equal to

(A) $\{(x, y, z) : x, y, z \in \mathbb{R}\}$
(B) $\{(0, y, z) : y, z \in \mathbb{R}\}$
(C) $\{(0,0,0)\}$
(D) $\{(0,0,z) : z \in \mathbb{R}\}$

124. A ray of light coming from origin after reflection at the point $P(x, y)$ of any curve becomes parallel to x-axis. The equation of the curve may be

(A) $y^2 = x$
(B) $y^2 = 2x+1$
(C) $y^2 = 4x$
(D) $y^2 = 4x+1$

125. The eccentricity of the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ is changed at the rate of $0.1 \text{units}/s$. The time at which it will touch the auxiliary circle is

(A) 2 s
(B) 3 s
(C) 5 s
(D) 6 s

126. The general solution of the differential equation $(2x - y + 1)\,dx + (2y - x + 1)\,dy = 0$ is

(A) $x^2 + y^2 + xy - x + y = c$
(B) $x^2 + y^2 - xy + x + y = c$
(C) $x^2 - y^2 + 2xy - x + y = c$
(D) $x^2 - y^2 - 2xy + x - y = c$

127. The value of $\Delta \tan^{-1}\left(\frac{n-1}{n}\right)$ with $h=1$ is equal to

(A) $\tan^{-1}\left(\frac{1}{2n^2}\right)$

(B) $\tan^{-1}\left(\frac{1}{2n}\right)$

(C) $\tan^{-1}\left(\frac{1}{n^2}\right)$

(D) $\tan^{-1}\left(\frac{1}{n}\right)$

128. The radius of the circle given by $|\vec{r}|=5$ and $\vec{r}.\left(\vec{i}+\vec{j}+\vec{k}\right)=3\sqrt{3}$ is

(A) 1
(B) 2
(C) 3
(D) 4

129. Consider the set $X=\{z\in\mathbb{C}:|z|>1\}\cup\{i\}$. Then X in $\mathbb{C}$ is

(A) open but not closed
(B) closed but not open
(C) neither open nor closed
(D) both open and closed

130. Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous map and $A=\{x\in\mathbb{R}:f(x)=0\}$. Then A is

(A) closed
(B) compact
(C) bounded
(D) open

131. The integral $\int_{|z|=2}\frac{\cos z}{z^3}dz$ equals

(A) πi
(B) $-\pi i$
(C) $2\pi i$
(D) $-2\pi i$

132. In $\mathbb{R}$ with usual metric, closure of $\mathbb{Z}$ is

(A) $\mathbb{Z}$
(B) ϕ
(C) $\mathbb{R}$
(D) $\mathbb{Q}$

133. Let $f(z) = \frac{1}{z}$. Then f is

(A) not continuous on $\{z \in C : 0 < |z| \le 1\}$

(B) continuous but not uniformly continuous on $\{z \in C : 0 < |z| \le 1\}$

(C) uniformly continuous on $\{z \in C : 0 < |z| \le 1\}$

(D) no where continuous

134. The set of discontinuities of function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = [x]$ ($[x]$ is the integral part of x), is

(A) $\mathbb{N}$
(B) ϕ
(C) $\mathbb{Q}$
(D) $\mathbb{Z}$

135. The value of $\int \frac{dz}{z+2}$, where $C : |z| = 1$, is

(A) 0
(B) $-\pi/2$
(C) $\pi/2$
(D) $2\pi i$

136. Let $A = \begin{pmatrix} 0 & \alpha \\ \beta & 0 \end{pmatrix}$ be such that $A^3 + A = 0$. Then

(A) $\alpha\beta = 2$
(B) $\alpha\beta \neq 1$
(C) $\alpha\beta = -1$
(D) $\alpha\beta \neq 0$

137. The angle between the radius vector and the tangent to the curve $r = a(1 - \cos\theta)$ at $\theta = \frac{\pi}{6}$

(A) $\frac{\pi}{12}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

138. Let V be the set of all 3×3 real matrices such that $A = (a_{ij})$ with $a_{11} + a_{22} + a_{33} = 0$. Then dimension of V as a real vector space is

(A) 3
(B) 7
(C) 8
(D) 9

139. The dimension of the subspace of R^3 spanned by $(-3, 0, 1)$, $(1, 2, 1)$ and $(3, 0, -1)$ is

(A) 3
(B) 2
(C) 1
(D) 0

140. The tangent at points on the curve $xy = 20$ which are parallel to the line $5x + y = 1$, are

(A) $(2,10),(2,-10)$
(B) $(2,10),(-2,-10)$
(C) $(10,2),(10,2)$
(D) $(3,4),(4,3)$

141. The function $f(x) = \begin{cases} 1, & x \in \mathbb{Q} \\ 0, & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$ is

(A) continuous everywhere
(B) continuous nowhere
(C) continuous at $x = 1$
(D) continuous at $x = 0$

142. In $V_3(\mathbb{R})$, let $v_1 = (1,0,1), v_2(1,3,1), v_3 = (3,2,1)$. Let $\{w_1, w_2, w_3\}$ be the orthonormal basis obtained from $\{v_1, v_2, v_3\}$ for $V_3(\mathbb{R})$. Then w_2 is

(A) $(0,3,0)$
(B) $(3,3,0)$
(C) $(1,3,1)$
(D) $(1,0,1)$

143. The series $g(z) = \sum_{n=1}^{\infty} \frac{1}{n^2} \sin n^2 \pi z$ is

(A) not absolutely convergent
(B) uniformly convergent
(C) convergent but not uniformly convergent
(D) no where convergent

144. $\lim_{x \to 0+} \log\left(\frac{1+x}{1-x}\right)^{\frac{1}{x}}$

(A) exists and is equal to 0
(B) exists and is equal to 1
(C) exists and is equal to 2
(D) does not exist

145. If $f(x) = 1 + \frac{3}{2 + \sin^2 x}$, then

(A) f is periodic with period $\frac{\pi}{2}$
(B) f is periodic with period π
(C) f is periodic with period 2π
(D) f is not periodic

146. The set of linearly independent solutions of the differential equation $\frac{d^4y}{dx^4} - \frac{d^2y}{dx^2} = 0$ is

(A) $\{1, x, e^x, e^{-x}\}$

(B) $\{1, x, e^{-x}, xe^{-x}\}$

(C) $\{1, x, e^x, xe^x\}$

(D) $\{1, x, e^x, xe^{-x}\}$

147. Which of the following function is uniformly continuous?

(A) $f(x) = \sin^2 x,\ x \in \mathbb{R}$

(B) $f(x) = \frac{1}{x},\ x \in (0,1)$

(C) $f(x) = x^2,\ x \in \mathbb{R}$

(D) $f(x) = x^2 + \frac{1}{x},\ x \in \mathbb{R}$

148. The orthogonal trajectories of the rectangular hyperbola $xy = a^2$ is

(A) $x^2 + y^2 = c^2$

(B) $x^2 = c^2 y^2$

(C) $x = c^2 y^2$

(D) $x^2 - y^2 = c^2$

149. The probability that a single toss of a die will result in a number less than 4 if the toss resulted in an odd number, is

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) 1

(D) $\frac{1}{2}$

150. If $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$, the sum and product of the eigen values of A are

(A) 32, 12
(B) 12, −32
(C) 12, 32
(D) −12, −32

FINAL ANSWER KEY

Subject Name: MATHEMATICS

SI No.	Key	SI No.	Key	SI No.	Key	SI No.	Key	SI No.	Key
1	B	31	C	61	A	91	C	121	B
2	B	32	C	62	B	92	A	122	B
3	A	33	A	63	D	93	D	123	D
4	B	34	D	64	A	94	A	124	B
5	D	35	D	65	C	95	C	125	C
6	A	36	D	66	D	96	B	126	B
7	D	37	A	67	B	97	B	127	A
8	B	38	A	68	A	98	C	128	D
9	A	39	B	69	B	99	D	129	C
10	A	40	B	70	D	100	C	130	A
11	D	41	A	71	A	101	C	131	B
12	D	42	C	72	A	102	B	132	A
13	C	43	B	73	C	103	B	133	B
14	C	44	C	74	D	104	A	134	D
15	B	45	A	75	B	105	A	135	A
16	C	46	C	76	D	106	C	136	C
17	D	47	B	77	A	107	B	137	A
18	C	48	B	78	D	108	D	138	C
19	A	49	A	79	B	109	B	139	B
20	A	50	C	80	D	110	D	140	B
21	B	51	D	81	A	111	B	141	B
22	B	52	B	82	C	112	B	142	A
23	B	53	C	83	C	113	D	143	B
24	B	54	C	84	B	114	D	144	C
25	C	55	C	85	B	115	C	145	B
26	D	56	A	86	D	116	B	146	A
27	C	57	A	87	A	117	A	147	A
28	A	58	D	88	B	118	A	148	D
29	D	59	A	89	C	119	C	149	B
30	D	60	C	90	B	120	B	150	C